

Textures and hierarchies in quark mass matrices with four texture-zeros

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Abstract

A systematic analysis on the phenomenologically allowed textures in Hermitian quark mass matrices with four texture-zeros is presented. The current data allow a full determination of the mass matrix elements, provided that the actual number of the parameters in the matrices is less than or equal to ten. Besides the standard texture with zeros located in (1,1) and (1,3) positions, we find four new type of parallel mass matrices. All of them are found to have similar hierarchical structures: three of the nonzero matrix elements are at the same order of magnitude with a small geometric hierarchy, while the remaining one is relatively much smaller. These textures show a possibility that a realistic quark mass matrix may have an approximate flavor $S(2)_L \otimes S(2)_R$ symmetry, i.e. a partial flavor democracy. The absolute values of the CP violating phases in the matrix elements are found to be either close to their maximum ($\approx 90^\circ$) or very small ($\leq 10^\circ$). We also find twenty-six quasi-parallel and seventeen non-parallel textures allowed by the experimental data.

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I. INTRODUCTION

Although the gauge sector of the standard model (SM) with $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry is a great success. The Yukawa sector of the SM is still poorly understood. The outstanding problems such as the origins of the fermion masses, the mixing angles as well as the CP violation remain to be the focus of the particle physics. There have been extensive studies on the possible underlying symmetries in the Yukawa coupling matrices in the SM, SUSY or other models (see, e.g.[1–16]). Lacking of a more fundamental theory of the basic interactions at present, the phenomenological model-independent approaches to searching for possible textures or symmetries in the fermion mass matrices are still playing important roles.

In the SM, the mass term in the Lagrangian is given by

$$-\mathcal{L}_M = \bar{u}_L M^u u_R + \bar{d}_L M^d d_R, \quad (1)$$

where the mass matrices M^u and M^d are three-dimensional complex matrices. In the most general case, they contain 36 real parameters, and are diagonalized through the following bi-unitary transformations

$$\begin{aligned} R_L^{u\dagger} M^u R_R^u &= \text{diag}\{m_u, m_c, m_t\}, \\ R_L^{d\dagger} M^d R_R^d &= \text{diag}\{m_d, m_s, m_b\}, \end{aligned} \quad (2)$$

with $R_{L,R}^{u,d}$ being the unitary rotation matrices for the left- and right-handed quark fields. The Cabibbo-Kobayashi-Maskawa (CKM) matrix V_{CKM} is then defined as the product of the left-handed rotation matrices for up and down quarks

$$V_{CKM} \equiv R^{u\dagger} R^d. \quad (3)$$

The observed large hierarchies in both the quark masses and the CKM matrix elements strongly imply that they may be related to each other through simple patterns or textures of the quark mass matrices. As it was noticed long time ago, the Cabibbo angle θ_C can be expressed in terms of quark mass ratio as $\theta_C \approx \sqrt{m_d/m_s}$ [17–19], which has aroused large amount of theoretical attempts in relating the CKM matrix elements to the quark mass ratios through certain textures of the quark mass matrices [1–9]. Among the theoretical activities, the models based on the texture-zeros have achieved some extent of success in phenomenology. A well known example in the two family case is a model with zeros in the (1,1) position for both up and down quark mass matrices, which correctly leads to the above mentioned value of the Cabibbo angle[1]. In the three family case, a straight forward extension is a model with six texture-zeros in total, often being referred to as Fritzsch model[2, 3]. In this model, the zeros are located in (1,1), (2,2) and (1,3) positions. This model can not only reproduce the same predictions for the Cabibbo angle but also give more relations to the other CKM matrix elements, such as $|V_{cb}| \approx |\sqrt{m_s/m_b} - \sqrt{m_c/m_t}e^{i\phi}|$ and $|V_{ub}/V_{cb}| \approx m_u/m_c$ etc.. The same texture was also applied to the multi-Higgs-doublet models and results in the widely used Cheng-Sher ansatz [20] (for recent discussions, see, e.g.[21, 22]). Other kinds of six-texture-zero pattern were also used in the $SU(5)$ grand unification theories to improve the predictions of mass relations between light fermions[23].

However, with the awareness of the possible heavy top quark mass, the six-texture-zero model was found to be disfavored. Phenomenological studies had indicated that the Fritzsch

model could not accommodate the large top quark mass of $m_t = 175\text{GeV}$ and small CKM matrix element of $|V_{cb}| \approx m_s/m_b \simeq 0.04$ simultaneously[24]. The current data actually rule out all the textures with six texture-zeros at electroweak scale[13]. If the up and down quark mass matrices are allowed to have different numbers of texture-zeros, the maximum number of zeros are five, and there are only five allowed patterns[13]. On the other hand, by requiring the up and down quark mass matrices have parallel structures, which is more symmetric and natural, one arrives at the models with four texture-zeros[25]. A four zeros texture with zeros located in (1,1) and (1,3) positions has been extensively discussed in the SM in the recent years(see, e.g. [6, 7, 26–29]). There are also attempts to use this texture in the $SO(10)$ grand unification theories [30, 31]. Recently, it has also been discussed in the general two-Higgs-doublet model, as it can lead to stronger suppressions of the flavor changing neutral currents associating with heavy fermions [32].

Compared with the textures with five or six texture-zeros, the models with four zeros has less predictability. At first glance, Hermitian mass matrices with four texture-zeros in (1,1) and (1,3) positions have 10 free parameters in total and can not give any prediction. It is not true if the hierarchy in the nonzero matrix elements can be obtained from symmetric considerations. For instance, the models with horizontal $U(2)$ symmetry [33, 34] or $D(3n^2)$ dihedral symmetry[30] will make the four-texture-zero textures have clear hierarchies in the remaining nonzero matrix elements. In those cases, the four-texture-zero model could be very predictive. With a hierarchy of $M^q(3,3) \gg M^q(2,2) \approx M^q(2,3) \gg M^q(1,2)$, such textures can give roughly correct predictions of $|V_{us}|$ and $|V_{cb}|$, etc..

Note that the current low energy data contain six quark masses and four independent CKM matrix elements. All of them are in reasonable precisions. Making use of these data, it is possible to determine *all* the nonzero matrix element in the four-texture-zero textures, provided that the actual number of free parameters is equal to or less than ten. This “bottom-up” approach, namely, constructing the possible textures of quark mass matrices from the low energy data has been applied to searching for the valid five zero textures before[13]. By constructing all the phenomenologically allowed textures from the data, one may find important hints of the underlying symmetries in the Yukawa sector. This approach is also suitable for discussing all the possible textures and hierarchies in the four-texture-zero models.

Currently, only one possibility of the four-texture-zero model has been carefully examined in the literature, namely, the texture-zeros located in (1,1) and (1,3) positions. This texture can be regarded as a simple extension of the Fritzsch model with (2,2) elements being nonzero. It respects the chiral evolution of quark masses and leads to a very useful parameterization of the CKM matrix [35, 36]. However, there is no any theoretical or experimental justification that this is the only valid texture with four-texture-zero. It is very likely that there exist much more valid ones with the four-texture-zero located in different places and with different hierarchies in the nonzero matrix elements. Those textures, once being constructed, might serve as a useful guide for model building.

The purpose of this paper is therefore to present a systematic analysis of the phenomenologically allowed textures of Hermitian quark mass matrices with four texture-zeros. It is organized as follows. In section **II**, we discuss the hierarchy in the standard four-texture-zero model with zeros located in (1,1) and (1,3) positions. we show that the nonzero elements of the quark mass matrices can be directly determined from the experimental data with a reasonable precision. In section **III**, we discuss all the four-texture-zero models with parallel structures, and find four new textures with zeros in other locations. In section **IV**, the valid

four-texture-zero models with quasi-parallel and non-parallel structure are discussed. We finally conclude in section V.

II. MASS MATRICES WITH TEXTURE ZEROS IN (1,1) AND (1,3) POSITIONS

In general, the three dimensional up and down quark mass matrices have totally 36 parameters, much more than the number of the observables. However, not all of them are physical. The ambiguities in the definition of the quark fields in the flavor basis allow the following unitary transformations which leave the physics unchanged

$$\begin{aligned} u_L &\rightarrow U_L u_L, & d_L &\rightarrow U_L d_L, \\ u_R &\rightarrow V_R^u u_R, & d_R &\rightarrow V_R^d d_R, \end{aligned} \quad (4)$$

where $U_L, V_R^{u(d)}$ are all unitary matrices. The quark mass matrices transform accordingly as

$$M^u \rightarrow U_L^\dagger M^u V_R^u, \quad M^d \rightarrow U_L^\dagger M^d V_R^d. \quad (5)$$

Without a loss of generality, one can arrive at physically equivalent mass matrices with less parameters by suitable transformations. For instance, by suitably choosing the unitary matrices $V_R^{u(d)}$ and U_L , the mass matrices $M^{u(d)}$ can always be arranged to be Hermitian[37]. Thus in all the bellow discussions, we shall focus only on the textures of Hermitian mass matrices.

By assuming that the matrix elements in (1,1) and (1,3) position are both vanishing, one arrives at the following Hermitian mass matrices with four texture-zeros, which is referred to as texture P1 [6, 7, 26–29]

(6)

Texture P1

$$M_q = \chi_q \begin{pmatrix} 0 & D_q e^{i\phi_{Dq}} & 0 \\ D_q e^{-i\phi_{Dq}} & C_q & B_q e^{i\phi_{Bq}} \\ 0 & B_q e^{-i\phi_{Bq}} & A_q \end{pmatrix}, \quad (q = u, d), \quad (7)$$

where A_q, B_q, C_q, D_q are real parameters χ_q is a scaling factor which is just the mass of the third family quarks, i.e. $\chi_{u(d)} = m_{t(b)}$. This texture can be rewritten as

$$M_q = \chi_q \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi_{Dq}} & 0 \\ 0 & 0 & e^{-i(\phi_{Bq} + \phi_{Dq})} \end{pmatrix} \begin{pmatrix} 0 & D_q & 0 \\ D_q & C_q & B_q \\ 0 & B_q & A_q \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_{Dq}} & 0 \\ 0 & 0 & e^{i(\phi_{Bq} + \phi_{Dq})} \end{pmatrix} \quad (8)$$

$$\equiv V_q^\dagger \widetilde{M}_q V_q, \quad (9)$$

with \widetilde{M}_q being a real symmetric matrix.

For Hermitian quark mass matrices, one can further perform a common unitary transformation on both left- and right-handed quark fields, which keeps the mass matrices to be

Hermitian and has no physical effect, namely, the physical observables are unchanged under the transformation of

$$M_q \rightarrow U^\dagger M_q U \quad (q = u, d). \quad (10)$$

Taking $U = V_u^\dagger$, one finds that the up quark mass matrix can always be rotated to be real, and only the phase differences between up and down quark mass matrix elements are physically relevant. It is then helpful to define two independent phases

$$\phi_1 \equiv \phi_{D_u} - \phi_{D_d}, \quad \text{and} \quad \phi_2 \equiv \phi_{B_u} - \phi_{B_d}. \quad (11)$$

Thus this type of mass matrix actually contains the following ten free parameters

$$A_{u(d)}, B_{u(d)}, C_{u(d)}, D_{u(d)}, \phi_1, \text{ and } \phi_2.$$

The eigenvalues of the matrix \widetilde{M}_q is denoted by $\lambda_{1q}, \lambda_{2q}$ and λ_{3q} , which are just the same as the quark masses m_{iq} up to a sign difference. As a phase convention, we require $\lambda_{3q} > 0$. Since $\det(\widetilde{M}_q) = -A_q D_q^2$, the value of λ_{1q} and λ_{2q} must have different signs. We then define $\lambda_{1q} = -\eta m_{1q}$ and $\lambda_{2q} = \eta m_{2q}$ with $\eta = \pm 1$ to distinguish the two cases. Making use of the transformation invariants such as $\text{Tr}(\widetilde{M})$, $\text{Tr}(\widetilde{M}^2)$ and $\det(\widetilde{M})$, one finds that the value of C_q , D_q and B_q depend only on the value of A_q

$$\begin{aligned} C_q &= \lambda_{1q} + \lambda_{2q} + \lambda_{3q} - A_q, \\ D_q &= \sqrt{-\frac{\lambda_{1q}\lambda_{2q}\lambda_{3q}}{A_q}}, \\ B_q &= \sqrt{-\frac{(A_q - \lambda_{1q})(A_q - \lambda_{2q})(A_q - \lambda_{3q})}{A_q}}. \end{aligned} \quad (12)$$

Thus the hierarchical structure of this type of mass matrix is completely determined by A_q .

The rotation matrix \widetilde{R}_q which diagonalize \widetilde{M}_q can be obtained by solving the characteristic equation and is found to be

$$\widetilde{R}_q \simeq \begin{pmatrix} 1 & \eta\varepsilon_q & \varepsilon_q^{3/2}\delta_q \\ -\eta\varepsilon_q(1 - \frac{1}{2}\delta_q^2) & 1 - \frac{1}{2}\delta_q^2 & -\delta_q \\ \eta\varepsilon_q\delta_q & -\delta_q & 1 - \frac{1}{2}\delta_q^2 \end{pmatrix}, \quad (13)$$

where we have used $|\lambda_{3q}|, A_q \gg |\lambda_{2q}| \gg |\lambda_{1q}|$ and the well known approximate relation of $|\lambda_{1q}/\lambda_{2q}| \simeq |\lambda_{2q}/\lambda_{3q}|$. One sees that the rotation matrix depends only on two variables ε_q and δ_q , which are defined through

$$\varepsilon_q^2 \equiv -\frac{\lambda_{1q}}{\lambda_{2q}} \quad \text{and} \quad \delta_q^2 \equiv \frac{\lambda_{3q} - A_q}{\lambda_{3q}}. \quad (14)$$

The CKM matrix is given by

$$\begin{aligned} V_{CKM} &= R_u^\dagger R_d \\ &= \widetilde{R}_u^T V_u V_d^\dagger \widetilde{R}_d \\ &= \widetilde{R}_u^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_1} & 0 \\ 0 & 0 & e^{i(\phi_1+\phi_2)} \end{pmatrix} \widetilde{R}_d. \end{aligned} \quad (15)$$

The absolute values of some of the CKM matrix elements are therefore given as follows (for $\eta = \pm 1$):

$$|V_{us}| \simeq \left| \varepsilon_d - \varepsilon_u \left(1 - \frac{\delta_d^2}{2} \right) \left(1 - \frac{\delta_u^2}{2} \right) e^{i\phi_1} - \delta_u \delta_d \varepsilon_u e^{i(\phi_1 + \phi_2)} \right|, \quad (16)$$

$$|V_{cb}| \simeq \left| \delta_d \left(1 - \frac{\delta_u^2}{2} \right) e^{i\phi_1} - \delta_u \left(1 - \frac{\delta_d^2}{2} \right) e^{i(\phi_1 + \phi_2)} \pm \delta_d \varepsilon_u \varepsilon_d^{3/2} \right|, \quad (17)$$

$$|V_{ub}| \simeq \left| \varepsilon_u \left[\delta_d \left(1 - \frac{\delta_u^2}{2} \right) e^{i\phi_1} - \delta_u \left(1 - \frac{\delta_d^2}{2} \right) e^{i(\phi_1 + \phi_2)} \right] \mp \delta_d \varepsilon_d^{3/2} \right|, \quad (18)$$

$$|V_{ts}| \simeq \left| \delta_d \left(1 - \frac{\delta_u^2}{2} \right) e^{i\phi_1} - \delta_u \left(1 - \frac{\delta_d^2}{2} \right) e^{i(\phi_1 + \phi_2)} \mp \delta_u \varepsilon_d \varepsilon_u^{3/2} \right|. \quad (19)$$

In a good approximation, the four zeros texture can reproduce the well known relation of

$$|V_{us}| \simeq |\varepsilon_d - \varepsilon_u e^{i\phi_1}| \quad \text{and} \quad |V_{cb}| \simeq |V_{ts}| = |\delta_d - \delta_u e^{i\phi_2}|. \quad (20)$$

But the ratio of $|V_{ub}/V_{cb}|$ may deviate from $\sqrt{m_u/m_c}$, as the corrections from the last terms in the expressions of $|V_{ub}|$ and $|V_{cb}|$ can not be ignored, which makes it to be able to accommodate the current data of $|V_{ub}/V_{cb}|$ that is slightly larger than $\sqrt{m_u/m_c}$ [38].

The above analytical expressions, although useful in understanding the general behavior of this type of mass matrices, can not directly determine the hierarchy in the nonzero matrix elements, since the values of δ_q or A_q are still unknown. A quantitative determination of all the matrix element can only be done through numerical methods. Since this texture contains only ten free parameters. It is then possible to fit all the parameters from the data on six quark masses and four independent CKM matrix elements. Through a global fit to the experimental data, one is able to answer two important questions: 1) Can this texture accommodate all the current data consistently? 2) What is the exact hierarchy in the matrix elements?

For the χ^2 fit, we take the input parameters of the quark masses mainly from Ref.[39]. But for strange quark masses, we shall adopt the recent update of $\bar{m}_s(2\text{GeV}) = 117.4 \pm 23.4$ MeV from Refs.[40, 41] which is a combined average of the strange quark mass from chiral perturbative theory and QCD spectrum sum rules. Using the renormalization group equation, we rescale all the quark masses to the energy scale of m_Z , which gives the following values

$$\begin{aligned} m_u(m_Z) &= (0.000883 - 0.00294) \text{ GeV}, \\ m_c(m_Z) &= (0.589 - 0.691) \text{ GeV}, \\ m_t(m_Z) &= (178 - 189) \text{ GeV}, \\ m_d(m_Z) &= (0.00177 - 0.0053) \text{ GeV}, \\ m_s(m_Z) &= (0.0553 - 0.0883) \text{ GeV}, \\ m_b(m_Z) &= (2.76 - 3.04) \text{ GeV}. \end{aligned} \quad (21)$$

We then choose four independent observables related to the CKM matrix elements. The values of the CKM matrix element $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ can be directly obtained from the measurement of semileptonic kaon and B meson decays. The value of $|V_{us}|$ and $|V_{cb}|$ have been measured with a good precision [39]

$$|V_{us}| = 0.2196 \pm 0.0023, \quad |V_{cb}| = 0.04 \pm 0.013 \pm 0.009. \quad (22)$$

The value of $|V_{ub}|$ is extracted from semileptonic decays $B \rightarrow \pi(\rho)\ell\nu$, which suffers from large theoretical uncertainties from the heavy quark effective theory. The latest data give [42, 43]

$$|V_{ub}| = \begin{cases} (4.09 \pm 0.59 \pm 0.69) \times 10^{-3} & (\text{LEP}), \\ (4.08 \pm 0.56 \pm 0.40) \times 10^{-3} & (\text{CLEO}). \end{cases} \quad (23)$$

There are many CP violating parameters, for example, the angle of the unitarity triangle α, β or γ . The angle β can be extracted from mixing induced CP violating processes such as $B \rightarrow J/\psi K_S$ without hadronic uncertainties, but there is no perfect method to cleanly extract the angles α and γ , as the interference between strong and weak phase is complicated. (for detailed discussions on the strong and weak phases in B decays, see e.g. [44–51]). Here, we use an other important CP violation parameter, the Jarlskog rephasing invariant CP violation quantity J_{CP} , defined through $J_{CP} \equiv \text{Im}(V_{ir}V_{js}V_{is}^*V_{jr}^*)$ [52, 53]. The value of J_{CP} from the recent global fit of the unitarity triangle is given by [54]

$$J_{CP} = (2.727 \pm 0.233) \times 10^{-5}. \quad (24)$$

Using the about data as inputs, the best fitted values of mass matrix elements in the four-texture-zero texture of Eq.(6) are found to be

$$(25)$$

Texture P1

$$M^u = \chi_u \begin{pmatrix} \mathbf{0} & (0.0001977 \pm 0.00004.989) & \mathbf{0} \\ \sim & (0.07398 \pm 0.03128) & (0.2555 \pm 0.05371) \\ \sim & \sim & (0.9295 \pm 0.03761) \end{pmatrix},$$

$$M^d = \chi_d \begin{pmatrix} \mathbf{0} & (0.005731 \pm 0.001515)e^{i(-87.5 \pm 36.27)^\circ} & \mathbf{0} \\ \sim & (0.1052 \pm 0.03922) & (0.2702 \pm 0.06171)e^{i(6.915 \pm 4.946)^\circ} \\ \sim & \sim & (0.9184 \pm 0.05849) \end{pmatrix}, \quad (26)$$

where the symbol “ \sim ” stands for the repeated or known matrix elements, since the matrices are Hermitian. From the fit results, we arrive at the following observations

1. A very consistent fit with the minimum χ^2 of $\chi_{min}^2 = 1.1 \times 10^{-5}$ is obtained, which clearly indicates that this texture is in a remarkable agreement with all the current data. Thus it is a good candidate for realistic quark mass matrices.
2. The value of A_q are determined in a good precision and are found to be

$$A_u = 0.9295 \pm 0.03761 \quad \text{and} \quad A_d = 0.9184 \pm 0.05849. \quad (27)$$

However, the deviation of A_q from unity is obvious. Accordingly, the value of δ_q s are

$$\delta_u = 0.2655 \pm 0.07083 \quad \text{and} \quad \delta_d = 0.2857 \pm 0.1023, \quad (28)$$

which are not very small. With these values the mass matrices in Eq.(25) can be written as the following approximate form

$$M^u = \chi_u \begin{pmatrix} \mathbf{0} & \varepsilon_u^3 & \mathbf{0} \\ \sim & \delta_u^2 & \delta_u \\ \sim & \sim & 1 - \delta_u^2 \end{pmatrix}, \quad M^d = \chi_d \begin{pmatrix} \mathbf{0} & \varepsilon_d^3 e^{i\phi_1} & \mathbf{0} \\ \sim & \delta_d^2 & \delta_d e^{i\phi_2} \\ \sim & \sim & 1 - \delta_d^2 \end{pmatrix}. \quad (29)$$

This is in strong disagreement with the texture proposed in Ref.[13], but agrees with the recent qualitative discussion in Ref.[38]. In both up and down quark mass matrices, comparing with the small (1,2) element of 5.7×10^{-3} (1.9×10^{-4}) for down (up) quark mass matrix, the (2,2), (2,3) and (3,3) elements are significantly larger and are roughly at the same order of magnitude. The ratios between (2,2), (2,3) and (3,3) elements are

$$\left| \frac{M^q(2,2)}{M^q(2,3)} \right| \simeq \left| \frac{M^q(2,3)}{M^q(3,3)} \right| \simeq \delta_q \simeq 0.3. \quad (30)$$

3. The absolute central value of one phase ϕ_1 is found to be close to 90° , i.e. $|\phi_1| \simeq 90^\circ$, which implies the possibility of the maximum CP violation [6, 55, 56], in the sense that the CP violating phase may take its maximum value in the real world. However, the error of about 40° is quite large. At present one can not draw a robust conclusion on that. The reason that the ϕ_1 is poorly determined can be understood from the expression of $|V_{us}|$ in Eq.(16), in which the ϕ_1 gives the interference between ε_d and ε_u . Since they differ by two orders of magnitude, the interference is rather weak and can not give enough information for the value of ϕ_1 . The other phase parameter ϕ_2 is found to be small ($\leq 10^\circ$) but nonzero.

III. FOUR ZEROS WITH PARALLEL TEXTURES

We proceed to discuss other possible textures with four texture-zeros. To this end, we first classify all the possible textures into three types, depending on the locations of the texture-zeros, namely,

1. Parallel textures. The texture-zeros have the same locations for both up and down quark mass matrices. There are totally 15 different textures which can be further divided into three catalogs: 1.a) Three textures with zeros both in off-diagonal positions. 1.b) Three textures with the texture-zeros both in diagonal positions. 1.c) Nine textures in which one zero is in diagonal position while the other one is off-diagonal.
2. Quasi-Parallel textures. For each quark mass matrix there exist two texture-zeros, but their locations are different for up and down quark mass matrices. There are 210 different textures in total.
3. Non-Parallel textures. One of the up and down mass matrices have three zeros while the other one has only one zero. There are 240 possibilities.

Among those textures, the mass matrices with parallel textures are of course the most simplest and symmetric ones, and therefore are of particular interest. Note that for the case 1.a of the parallel texture, there are 9 free parameters, i.e. four matrix elements for

each quark mass matrix and one relative phase. The number of parameters is less than that of the observables. Thus they are over determined. For case 1.b there are totally 12 free parameters, more than the number of the available data. So, the matrix elements of these textures remain to be undetermined and will not be discussed here. For case 1.c, there are 10 parameters in total. In principle all the matrix elements can be determined.

We adopt the χ^2 fit method to find all the phenomenologically allowed mass matrices with four texture-zeros and parallel textures. To this end, a selection criterion is needed. In the fits we shall exclude all the textures which lead to any of the best fitted values of the quark masses, the CKM matrix elements $|V_{us}|$, $|V_{ub}|$, $|V_{cb}|$ or the J_{CP} outside the 1σ experimentally allowed ranges in Eqs.(21), (22), (23) and (24). In the parallel textures, by using Eq.(10), one can always rotate the up quark mass matrix to be real. We then choose it as a phase convention. The fit results show four phenomenologically allowed textures other than the texture P1, being referred to as texture P2-P5, These textures as well as the best fitted matrix elements are as follows:

1) Texture P2: Zeros are located in (1,1) and (1,2) positions. The best fitted up and down quark mass matrices are

Texture P2

$$M^u = \chi_u \begin{pmatrix} \mathbf{0} & \mathbf{0} & (0.000279 \pm 0.00007289) \\ \sim & (0.467 \pm 0.01485) & (0.497 \pm 0.006079) \\ \sim & \sim & (0.5365 \pm 0.01672) \end{pmatrix},$$

$$M^d = \chi_d \begin{pmatrix} \mathbf{0} & \mathbf{0} & (0.007886 \pm 0.0008101)e^{i(87.31 \pm 43.98)^\circ} \\ \sim & (0.4851 \pm 0.03067) & (0.4875 \pm 0.02665)e^{i(-175.5 \pm 0.3352)^\circ} \\ \sim & \sim & (0.5385 \pm 0.03412) \end{pmatrix}, \quad (31)$$

with a minimum χ^2 of $\chi_{min}^2 = 1.6 \times 10^{-6}$ which indicates a very consistent fit. From the numerical values, one sees that in this texture the elements associating with the second and the third families, i.e. the elements in (2,2), (2,3) and (3,3) positions could be very close to each other. The ratios between them are given by

$$\left| \frac{M^q(2,2)}{M^q(2,3)} \right| \simeq \left| \frac{M^q(2,3)}{M^q(3,3)} \right| \simeq 1, \quad (q = u, d). \quad (32)$$

Thus in a good approximation this texture has the following simple form

$$M^q \simeq \frac{\chi_q}{2} \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & \mathcal{O}(\varepsilon_q^3) \\ 0 & 0 & 0 \\ \mathcal{O}(\varepsilon_q^3) & 0 & 0 \end{pmatrix} \right], \quad (q = u, d). \quad (33)$$

It was noticed long ago that the large hierarchy in the quark masses suggests that the realistic quark mass matrices are close to a "democracy" limit [3, 5]

$$M^q \approx \frac{\chi_q}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (q = u, d), \quad (34)$$

which poses a flavor $S(3)_L \otimes S(3)_R$ symmetry. To generate the quark masses of the first and the second families and also the mixing angles, such a symmetry must be broken down. A popular way is to break the symmetry through the steps of $S(3)_L \otimes S(3)_R \rightarrow S(2)_L \otimes S(2)_R \rightarrow \times$, (see, e.g. [5, 57, 58]), where the $S(2)_L \otimes S(2)_R$ is associated with the first and the second family quarks. However, the texture P2 found here shows an example that the symmetry of $S(3)_L \otimes S(3)_R$ may be firstly broken down to a $S(2)_L \otimes S(2)_R$ symmetry associating with the second and the third families, and then the $S(2)_L \otimes S(2)_R$ symmetry is slightly broken down by a tiny perturbation to the (1,3) element. The small value of $M^q(1,3)/M^q(2,2) \approx \mathcal{O}(10^{-2} \sim 10^{-3})$ indicates the symmetry breaking strength. Thus the realistic quark mass matrix could have an approximate $S(2)_L \otimes S(2)_R$ symmetry, namely, a partial flavor democracy. In this texture, one also find a large phase of 87° which is close to the maximum (90°) in (1,3) element of the down quark mass matrix. The possibility of the maximum phase therefore does not uniquely belong to the texture P1. It is of interest to note that starting from such a partial flavor democracy some interesting relations between the quark masses and the mixing angles can be correctly reproduced [59].

2) Texture P3: Zeros are located in (1,2) and (2,2) positions. The best fitted matrix elements are found to be

Texture P3

$$M^u = \chi_u \begin{pmatrix} (0.3788 \pm 0.03425) & \mathbf{0} & (0.4828 \pm 0.01464) \\ \sim & \mathbf{0} & (0.0003199 \pm 0.00007) \\ \sim & \sim & (0.6243 \pm 0.03724) \end{pmatrix},$$

$$M^d = \chi_d \begin{pmatrix} (0.4244 \pm 0.04044) & \mathbf{0} & (0.4829 \pm 0.02282)e^{i(-2.756 \pm 0.4887)^\circ} \\ \sim & \mathbf{0} & (0.006209 \pm 0.0005847)e^{i(-176 \pm 10.51)^\circ} \\ \sim & \sim & (0.6042 \pm 0.04269) \end{pmatrix}. \quad (35)$$

with a minimum χ^2 of $\chi_{min}^2 = 0.86$. The numerical values show that the (1,1) (1,3) and (3,3) elements are at the same order of magnitude with a small geometric hierarchy of

$$\left| \frac{M^q(1,1)}{M^q(1,3)} \right| \approx \left| \frac{M^q(1,3)}{M^q(3,3)} \right| \approx 0.7, \quad (q = u, d). \quad (36)$$

Therefore it can be roughly understood as another type of partial flavor democracy between the first and the third families. On the other hand, the (2,3) elements are found to be much smaller. In this texture, the quark masses of the second family arise purely from the mixing terms. A small phase of a few degrees in (1,3) element and a phase around $\sim -180^\circ$ in the (2,3) element in down quark mass matrix are found. It shows however that the quark mass matrices are nearly real. Considering the fact that the experimentally observed effects of CP violation is quite small, this texture with small imaginary matrix elements is also natural.

3) Texture P4. Zeros in (2,2) and (2,3) elements. The best fitted mass matrices are

Texture P4

$$M^u = \chi_u \begin{pmatrix} (0.07226 \pm 0.01143) & (0.0001978 \pm 0.0000486) & (0.2526 \pm 0.01951) \\ \sim & \mathbf{0} & \mathbf{0} \\ \sim & \sim & (0.9313 \pm 0.03255) \end{pmatrix},$$

$$M^d = \chi_d \begin{pmatrix} (0.103 \pm 0.02048) & (0.005712 \pm 0.001497)e^{i(88.43 \pm 35.07)^\circ} & (0.2669 \pm 0.03301)e^{i(8.9 \pm 4.354)^\circ} \\ \sim & \mathbf{0} & \mathbf{0} \\ \sim & \sim & (0.9204 \pm 0.04911) \end{pmatrix}. \quad (37)$$

with $\chi_{min}^2 = 4.5 \times 10^{-4}$. One sees that (1,1) (1,3) and (3,3) elements are roughly at the same order of magnitudes with a geometric hierarchy of

$$\left| \frac{M^q(1,1)}{M^q(1,3)} \right| \approx \left| \frac{M^q(1,3)}{M^q(3,3)} \right| \approx 0.3, \quad (q = u, d). \quad (38)$$

while the (1,2) element are extremely small. In this texture, the quark masses of the second family also come from mixing terms. The phase of the (1,2) element is large and close to 90° .

4) Texture P5. Zeros in (2,3) and (3,3) elements. The best fitted up and down quark mass matrices are given by

Texture P5

$$M^u = \chi_u \begin{pmatrix} (0.06263 \pm 0.001382) & (0.2489 \pm 0.002558) & (0.0001981 \pm 0.00002745) \\ \sim & (0.934 \pm 0.01647) & \mathbf{0} \\ \sim & \sim & \mathbf{0} \end{pmatrix} \quad (39)$$

$$M^d = \chi_d \begin{pmatrix} (0.0980 \pm 0.00313) & (0.259 \pm 0.00565)e^{i(7.79 \pm 1.23)^\circ} & (0.00567 \pm 0.000632)e^{i(-90.3 \pm 17.3)^\circ} \\ \sim & (0.9255 \pm 0.02941) & \mathbf{0} \\ \sim & \sim & \mathbf{0} \end{pmatrix} \quad (40)$$

with $\chi_{min}^2 = 3 \times 10^{-3}$. Clearly, there is a similar geometric hierarchy in the (1,1), (1,2) and (2,2) elements:

$$\left| \frac{M^q(1,1)}{M^q(1,2)} \right| \approx \left| \frac{M^q(1,2)}{M^q(2,2)} \right| \approx 0.3. \quad (41)$$

Unlike all the other textures with large (3,3) elements, it has zeros in (3,3) element while a large (2,2) element with $M(2,2) \simeq 1$. In this type of mass matrices, the large third family quark masses are purely generated from small mixing terms. This mismatched texture can not be generated from the chiral evolutions of the quark mass matrices. One sees again that the smallest matrix element in (1,3) position is accompanied with a large phase with the absolute value around 90° .

In summary, besides the standard four zero textures, there are totally four phenomenologically allowed quark mass matrices with parallel textures. These textures, although differ

in the location of the texture-zeros, show similar hierarchical structures: three of the elements are roughly at order one with a small geometrical hierarchy, and the other one is much smaller. It indicates that some of those textures may be described by a partial flavor democracy for the related families, and the small elements may have a perturbative origin. The large CP violating phases with the absolute value around its maximum ($\sim 90^\circ$) are found in texture P1, P2, P4 and P5. But in texture P3, the matrix elements are almost real. Thus the values of the CP violating phases strongly depend on the locations of the texture-zeros.

IV. QUASI-PARALLEL AND NON-PARALLEL TEXTURES

If one abandons the parallel condition, within the framework of the four texture-zeros, much more phenomenologically allowed textures can be constructed. As it is mentioned in the previous sections, those textures can be roughly divided into two types: quasi-parallel and non-parallel textures.

For quasi-parallel textures, the up and down quark mass matrices may have two zeros in different places. If both of the mass matrices have two zeros in the off-diagonal positions there will be 8 parameters left. If one of the quarks mass matrices have two zeros in the off-diagonal positions while the other one have only one zero in the off-diagonal position, there will be 9 free parameters. In this case, the mass matrix which has one zero in the off-diagonal position can be rotated to be real by a suitable unitary transformation from Eq.(10). If both of the mass matrices have one off-diagonal zero, the number of free parameters is 10. One can just take the up quark mass matrix to be real as in the section II. Thus, in all the above cases, the matrix elements can be determined by the experimental data. A direct search shows that there are 26 allowed quasi-parallel textures. Those textures as well as the best fitted matrix elements are summarized in appendix A.

Due to the much larger hierarchy in the up quark masses than that in the down quark masses, compared with textures in the up quark mass matrices, the one in the down quark mass matrix plays a more fundamental role in determining the CKM matrix elements. This down quark dominance partially explains why there exist many allowed quasi-parallel textures with four texture-zeros: for certain type of down quark mass matrix, there could be more than one type of phenomenologically allowed up quark mass matrices. For instance, the fit results show that there are six other allowed textures in which the zeros are located in (1,1) and (1,3) positions in down quark mass matrix but the zeros are in different places in the up quark mass matrix. The related best fits of the matrix elements are listed in Eqs. (A1), (A8),(A11),(A15),(A18),(A20) in the appendix A. A typical one in Eq.(A18) is given by

$$M^u = \chi_u \begin{pmatrix} (-0.00001738 \pm 0.00001811) & (0.0003112 \pm 0.00009818) & \mathbf{0} \\ \sim & \mathbf{0} & (-0.05882 \pm 0.002509) \\ \sim & \sim & (0.9965 \pm 0.02998) \end{pmatrix},$$

$$M^d = \chi_d \begin{pmatrix} \mathbf{0} & (0.005366 \pm 0.001217)e^{i(93.5 \pm 14.31)^\circ} & \mathbf{0} \\ \sim & (0.03348 \pm 0.005211) & (0.09573 \pm 0.009601)e^{i(-177.4 \pm 73.63)^\circ} \\ \sim & \sim & (0.9904 \pm 0.04784) \end{pmatrix}.$$

In this texture, although the up quark mass matrices are significantly different than that in the texture P1, the down quark mass matrices show a similar hierarchy, and the absolute value of the phase in (1,3) element is also close to 90° .

Similarly, there are three more textures (see Eqs. (A5),(A10) and (A22)) in which the zeros are located in (1,1) and (1,2) positions in down quark mass matrices. For example in Eq.(A10), the following mass matrix is allowed

$$M^u = \chi_u \begin{pmatrix} (-0.00001042 \pm 0.000005605) & \mathbf{0} & \mathbf{0} \\ \sim & (0.7212 \pm 0.09639) & (0.4473 \pm 0.048)e^{i(-4.518 \pm 0.6164)^\circ} \\ \sim & \sim & (0.2822 \pm 0.09429) \end{pmatrix},$$

$$M^d = \chi_d \begin{pmatrix} \mathbf{0} & \mathbf{0} & (0.006616 \pm 0.001082) \\ \sim & (0.697 \pm 0.09951) & (0.4672 \pm 0.04813) \\ \sim & \sim & (0.2796 \pm 0.104) \end{pmatrix},$$

where, the partial democratic form in the down quark mass matrix still remains to some extent, despite a different form of the up quark mass matrix.

For non-parallel textures, i.e., one of the up and down quark mass matrices have three texture zeros, the maximum number of parameter are usually less than or equal to 10, except for the ones with totally three zeros in diagonal position. For completeness, we also perform a primitive search for the valid textures and find 15 allowed types. This type of texture is of less interest as it is less symmetric, and has not yet been considered in the model constructions. The details of those allowed textures of this type will not be discussed there. However, note that although the Fritzsch model with six texture zeros in (1,1), (1,3) and (2,2) positions in both up and down quark mass matrices have been ruled out, the down quark mass matrix can have three zeros in such positions alone. For example, the following quark mass matrix is found to be allowed

$$M^u = \chi_u \begin{pmatrix} (0.000005803 \pm 0.00003106) & \mathbf{0} & (0.001197 \pm 0.00138) \\ \sim & (0.03942 \pm 0.01812) & (0.1858 \pm 0.04586) \\ \sim & \sim & (0.964 \pm 0.03157) \end{pmatrix},$$

$$M^d = \chi_d \begin{pmatrix} \mathbf{0} & (-0.005556 \pm 0.001399)e^{i(95.08 \pm 50.67)^\circ} & \mathbf{0} \\ \sim & \mathbf{0} & (0.1535 \pm 0.01946)e^{i(3.515 \pm 45.09)^\circ} \\ \sim & \sim & (0.9764 \pm 0.04894) \end{pmatrix},$$

with a large phase of $\phi = 95^\circ$ in the (1,2) elements of the down quark mass matrix.

V. CONCLUSIONS

In conclusion, we have systematically explored all the possible four zero textures with the number of parameters less than or equal to ten. We have found that there are totally five allowed mass matrices with parallel textures. It is shown that the phenomenologically allowed mass matrices should have one relatively small matrix elements while three others are at the same order of magnitude with a small geometric hierarchy. The hierarchies in the mass matrices can be written in the following approximate form

$$\begin{matrix} \text{P1} & \text{P2} & \text{P3} & \text{P4} & \text{P5} \\ \begin{pmatrix} 0 & \epsilon_1 & 0 \\ \epsilon_1^* & \lambda_1^2 & \lambda_1 \\ 0 & \lambda_1 & 1 \end{pmatrix}, & 0.5 \begin{pmatrix} 0 & 0 & \epsilon_2 \\ 0 & \lambda_2^2 & \lambda_2 \\ \epsilon_2^* & \lambda_2 & 1 \end{pmatrix}, & 0.6 \begin{pmatrix} \lambda_3^2 & 0 & \lambda_3 \\ 0 & 0 & \epsilon_3 \\ \lambda_3 & \epsilon_3^* & 1 \end{pmatrix}, & \begin{pmatrix} \lambda_4^2 & \epsilon_4 & \lambda_4 \\ \epsilon_4^* & 0 & 0 \\ \lambda_4 & 0 & 1 \end{pmatrix}, & \begin{pmatrix} \lambda_5^2 & \lambda_5 & \epsilon_5 \\ \lambda_5 & 1 & 0 \\ \epsilon_5^* & 0 & 0 \end{pmatrix}, \end{matrix} \quad (42)$$

with

$$\lambda_1 \simeq \lambda_4 \simeq \lambda_5 \simeq 0.3, \quad \lambda_2 \simeq 1.0, \quad \text{and} \quad \lambda_3 \simeq 0.7. \quad (43)$$

The value of ϵ_i s are much smaller, i.e., $|\epsilon_i| \ll |\lambda_i|$. Large phases in ϵ_i with the absolute values closing to 90° are found for texture P1, P2, P4 and P5. But in texture P3 all the phases are found to be small. We have also found 26 quasi-parallel textures and 17 non-parallel textures. A partial flavor democracy is observed in the texture P2, which implies that an approximate flavor permutation symmetry of $S(2)_L \otimes S(2)_R$ could remain after the breaking of $S(3)_L \otimes S(3)_R$. This may serve as a guide for the model buildings. The textures found there may also be helpful in discussing the related problems. For example, it remains to be seen whether those textures can also be applied to the lepton mass matrixes in the SM and the Yukawa coupling matrices with multi-Higgs-doublet models. It could also be helpful to search the similar textures in the Yukawa sector of various grand unification theories at high energy scale.

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APPENDIX A: PHENOMENOLOGICALLY ALLOWED MASS MATRICES WITH QUASI-PARALLEL TEXTURES

1.

$$M^u = \chi_u \begin{pmatrix} \mathbf{0} & \mathbf{0} & (0.002992 \pm 0.001939) \\ \sim & (0.004123 \pm 0.004289) & (-0.02518 \pm 0.08469) \\ \sim & \sim & (0.9993 \pm 0.0302) \end{pmatrix}$$

$$M^d = \chi_d \begin{pmatrix} \mathbf{0} & (0.005557 \pm 0.001228)e^{i(-84.19 \pm 61.65)^\circ} & \mathbf{0} \\ \sim & (0.02388 \pm 0.005598) & (0.02384 \pm 0.03518)e^{i(72.52 \pm 245.1)^\circ} \\ \sim & \sim & (0.9994 \pm 0.04827) \end{pmatrix} \quad (A1)$$

2.

$$M^u = \chi_u \begin{pmatrix} \mathbf{0} & \mathbf{0} & (0.002679 \pm 0.0007222) \\ \sim & (0.005071 \pm 0.0003244) & (0.03973 \pm 0.002328) \\ \sim & \sim & (0.9984 \pm 0.02993) \end{pmatrix}$$

$$M^d = \chi_d \begin{pmatrix} \mathbf{0} & (0.005389 \pm 0.00149)e^{i(-79.17 \pm 111.6)^\circ} & (0.006903 \pm 0.002963)e^{i(22.01 \pm 52.48)^\circ} \\ \sim & (0.02361 \pm 0.005742) & \mathbf{0} \\ \sim & \sim & (1 \pm 0.04827) \end{pmatrix} \quad (A2)$$

3.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} \mathbf{0} & \mathbf{0} & (-0.00101 \pm 0.00008551) \\ \sim & (0.03584 \pm 0.001073) & (0.1766 \pm 0.002824) \\ \sim & \sim & (0.9676 \pm 0.02039) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} (0.000232 \pm 0.000588) & (-0.00503 \pm 0.000445)e^{i(-112 \pm 17.5)^\circ} & \mathbf{0} \\ \sim & \mathbf{0} & (0.154 \pm 0.008)e^{i(-10 \pm 3.1)^\circ} \\ \sim & \sim & (0.976 \pm 0.0426) \end{pmatrix}
\end{aligned} \tag{A3}$$

4.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} \mathbf{0} & \mathbf{0} & (0.002473 \pm 0.0001303) \\ \sim & (0.005087 \pm 0.0003234) & (0.0399 \pm 0.002323) \\ \sim & \sim & (0.9987 \pm 0.02991) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} (0.002504 \pm 0.0006562) & (0.005346 \pm 0.001301)e^{i(123.4 \pm 10.61)^\circ} & \mathbf{0} \\ \sim & (0.02347 \pm 0.005364) & \mathbf{0} \\ \sim & \sim & (1 \pm 0.04828) \end{pmatrix}
\end{aligned} \tag{A4}$$

5.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} \mathbf{0} & (0.0004591 \pm 0.00006657) & \mathbf{0} \\ \sim & (0.8188 \pm 0.0305) & (-0.3843 \pm 0.009781) \\ \sim & \sim & (0.1846 \pm 0.01171) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} \mathbf{0} & \mathbf{0} & (0.005938 \pm 0.001072)e^{i(-87.66 \pm 11.46)^\circ} \\ \sim & (0.8518 \pm 0.04308) & (0.3504 \pm 0.02167)e^{i(-180 \pm 0.01061)^\circ} \\ \sim & \sim & (0.1718 \pm 0.01536) \end{pmatrix}
\end{aligned} \tag{A5}$$

6.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} \mathbf{0} & (0.0001908 \pm 0.00005187) & \mathbf{0} \\ \sim & (0.005055 \pm 0.0003245) & (-0.03962 \pm 0.002329) \\ \sim & \sim & (0.9984 \pm 0.02992) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} \mathbf{0} & (0.005559 \pm 0.00151)e^{i(-96.31 \pm 59.68)^\circ} & (0.005524 \pm 0.001065)e^{i(21.95 \pm 39.94)^\circ} \\ \sim & (-0.02351 \pm 0.005726) & \mathbf{0} \\ \sim & \sim & (0.9999 \pm 0.04827) \end{pmatrix}
\end{aligned} \tag{A6}$$

7.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} \mathbf{0} & (0.05503 \pm 0.002538) & \mathbf{0} \\ \sim & (0.9958 \pm 0.02998) & (-0.02169 \pm 0.004031) \\ \sim & \sim & (-0.00001203 \pm 0.000006505) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} (0.0150 \pm 0.00606) & (0.094 \pm 0.00537)e^{i(-180 \pm 2.37)^\circ} & \mathbf{0} \\ \sim & (-0.993 \pm 0.0479) & (0.0385 \pm 0.00745)e^{i(-5.23 \pm 2.8)^\circ} \\ \sim & \sim & \mathbf{0} \end{pmatrix}
\end{aligned} \tag{A7}$$

8.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} \mathbf{0} & (-0.00002781 \pm 0.00009973) & (0.003193 \pm 0.001108) \\ \sim & (0.003487 \pm 0.0002779) & \mathbf{0} \\ \sim & \sim & (1 \pm 0.02997) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} \mathbf{0} & (0.005497 \pm 0.001175)e^{i(67.18 \pm 71.37)^\circ} & \mathbf{0} \\ \sim & (0.02511 \pm 0.00494) & (0.03905 \pm 0.002748)e^{i(-180 \pm 59.59)^\circ} \\ \sim & \sim & (0.9984 \pm 0.0482) \end{pmatrix}
\end{aligned} \tag{A8}$$

9.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} \mathbf{0} & (0.0003036 \pm 0.00002996) & (-0.05884 \pm 0.002505) \\ \sim & (-0.00003703 \pm 0.000006764) & \mathbf{0} \\ \sim & \sim & (0.9965 \pm 0.02997) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} (0.0206 \pm 0.00449) & (-0.00603 \pm 0.00127)e^{i(50 \pm 10.4)^\circ} & (0.0184 \pm 0.00327)e^{i(-180 \pm 37)^\circ} \\ \sim & \mathbf{0} & \mathbf{0} \\ \sim & \sim & (0.999 \pm 0.0483) \end{pmatrix}
\end{aligned} \tag{A9}$$

10.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (-0.00001042 \pm 0.000005605) & \mathbf{0} & \mathbf{0} \\ \sim & (0.7212 \pm 0.09639) & (0.4473 \pm 0.048)e^{i(-4.518 \pm 0.6164)^\circ} \\ \sim & \sim & (0.2822 \pm 0.09429) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} \mathbf{0} & \mathbf{0} & (0.006616 \pm 0.001082) \\ \sim & (0.697 \pm 0.09951) & (0.4672 \pm 0.04813) \\ \sim & \sim & (0.2796 \pm 0.104) \end{pmatrix}
\end{aligned} \tag{A10}$$

11.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (0.00001042 \pm 0.000005605) & \mathbf{0} & \mathbf{0} \\ \sim & (0.2833 \pm 0.06691) & (0.4478 \pm 0.03507)e^{i(-4.513 \pm 0.5086)^\circ} \\ \sim & \sim & (0.7202 \pm 0.06983) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} \mathbf{0} & (0.006604 \pm 0.0007225) & \mathbf{0} \\ \sim & (0.3135 \pm 0.07092) & (0.4463 \pm 0.04499) \\ \sim & \sim & (0.7098 \pm 0.06513) \end{pmatrix}
\end{aligned} \tag{A11}$$

12.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (0.00001042 \pm 0.000005605) & \mathbf{0} & \mathbf{0} \\ \sim & (0.005081 \pm 0.0003235) & (0.03982 \pm 0.00232)e^{i(-117.8 \pm 11.23)^\circ} \\ \sim & \sim & (0.9984 \pm 0.02993) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} \mathbf{0} & (0.005536 \pm 0.001156) & (0.0036 \pm 0.0002005) \\ \sim & (0.02334 \pm 0.004875) & \mathbf{0} \\ \sim & \sim & (1 \pm 0.04827) \end{pmatrix}
\end{aligned} \tag{A12}$$

13.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (-0.001891 \pm 0.0003225) & \mathbf{0} & (-0.03989 \pm 0.00231) \\ \sim & \mathbf{0} & (-0.004382 \pm 0.001207) \\ \sim & \sim & (0.9984 \pm 0.02992) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} (0.0235 \pm 0.0057) & (0.00549 \pm 0.00149)e^{i(-87.6 \pm 64.1)^\circ} & \mathbf{0} \\ \sim & \mathbf{0} & (0.00153 \pm 0.00207)e^{i(-46.7 \pm 136.9)^\circ} \\ \sim & \sim & (1 \pm 0.0483) \end{pmatrix}
\end{aligned} \tag{A13}$$

14.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (0.004253 \pm 0.004838) & \mathbf{0} & (-0.02764 \pm 0.08707) \\ \sim & \mathbf{0} & (0.002949 \pm 0.002012) \\ \sim & \sim & (0.9992 \pm 0.03032) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} (0.0239 \pm 0.00573) & (0.00556 \pm 0.00124)e^{i(84.8 \pm 62.4)^\circ} & (0.0241 \pm 0.0373)e^{i(80.4 \pm 180)^\circ} \\ \sim & \mathbf{0} & \mathbf{0} \\ \sim & \sim & (0.999 \pm 0.0483) \end{pmatrix}
\end{aligned} \tag{A14}$$

15.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (0.00000336 \pm 0.00000569) & \mathbf{0} & (0.00371 \pm 0.000163)e^{i(59.2 \pm 11.0)^\circ} \\ \sim & (0.00349 \pm 0.000278) & \mathbf{0} \\ \sim & \sim & (1 \pm 0.03) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} \mathbf{0} & (0.005547 \pm 0.001157) & \mathbf{0} \\ \sim & (0.02495 \pm 0.004871) & (0.03903 \pm 0.002749) \\ \sim & \sim & (0.9985 \pm 0.0482) \end{pmatrix}
\end{aligned} \tag{A15}$$

16.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (0.2771 \pm 0.1022) & \mathbf{0} & (0.4504 \pm 0.0518)e^{i(4.519 \pm 0.6441)^\circ} \\ \sim & (0.00001042 \pm 0.000005605) & \mathbf{0} \\ \sim & \sim & (0.7195 \pm 0.1041) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} (0.2794 \pm 0.1118) & (0.006618 \pm 0.001157) & (0.4671 \pm 0.05117) \\ \sim & \mathbf{0} & \mathbf{0} \\ \sim & \sim & (0.6971 \pm 0.1063) \end{pmatrix}
\end{aligned} \tag{A16}$$

17.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (-0.003488 \pm 0.0002779) & \mathbf{0} & (-0.00003017 \pm 0.0001379) \\ \sim & (1 \pm 0.02997) & (0.003267 \pm 0.0007881) \\ \sim & \sim & \mathbf{0} \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} (-0.0219 \pm 0.00552) & (0.0409 \pm 0.00282)e^{i(50.4 \pm 308.7)^\circ} & (-0.0055 \pm 0.00143)e^{i(109.4 \pm 268.5)^\circ} \\ \sim & (0.9984 \pm 0.0482) & \mathbf{0} \\ \sim & \sim & \mathbf{0} \end{pmatrix}
\end{aligned} \tag{A17}$$

18.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (-0.00001738 \pm 0.00001811) & (0.0003112 \pm 0.00009818) & \mathbf{0} \\ \sim & \mathbf{0} & (-0.05882 \pm 0.002509) \\ \sim & \sim & (0.9965 \pm 0.02998) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} \mathbf{0} & (0.005366 \pm 0.001217)e^{i(93.5 \pm 14.31)^\circ} & \mathbf{0} \\ \sim & (0.03348 \pm 0.005211) & (0.09573 \pm 0.009601)e^{i(-177.4 \pm 73.63)^\circ} \\ \sim & \sim & (0.9904 \pm 0.04784) \end{pmatrix}
\end{aligned} \tag{A18}$$

19.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (0.003488 \pm 0.0002664) & (-0.00002778 \pm 0.00002023) & \mathbf{0} \\ \sim & \mathbf{0} & (0.003194 \pm 0.0002309) \\ \sim & \sim & (1 \pm 0.02734) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} (0.0251 \pm 0.00109) & (0.0055 \pm 0.000254)e^{i(-67.2 \pm 111.2)^\circ} & (0.0391 \pm 0.00262)e^{i(-180 \pm 112.1)^\circ} \\ \sim & \mathbf{0} & \mathbf{0} \\ \sim & \sim & (0.998 \pm 0.0419) \end{pmatrix}
\end{aligned} \tag{A19}$$

20.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (0.00001761 \pm 0.000006948) & (0.0003119 \pm 0.00003149)e^{i(-88.95 \pm 12.3)^\circ} & \mathbf{0} \\ \sim & (0.00346 \pm 0.0002757) & \mathbf{0} \\ \sim & \sim & (1 \pm 0.02997) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} \mathbf{0} & (-0.005197 \pm 0.001161) & \mathbf{0} \\ \sim & (0.02587 \pm 0.00507) & (0.03907 \pm 0.002734) \\ \sim & \sim & (0.9984 \pm 0.0482) \end{pmatrix}
\end{aligned} \tag{A20}$$

21.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (0.003457 \pm 0.0002755) & (0.0003266 \pm 0.00003135)e^{i(-46.19 \pm 10.07)^\circ} & \mathbf{0} \\ \sim & (0.00002034 \pm 0.000006928) & \mathbf{0} \\ \sim & \sim & (1 \pm 0.02997) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} (0.02102 \pm 0.00424) & (0.006014 \pm 0.001289) & (0.03921 \pm 0.002804) \\ \sim & \mathbf{0} & \mathbf{0} \\ \sim & \sim & (0.9987 \pm 0.04818) \end{pmatrix}
\end{aligned} \tag{A21}$$

22.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (-0.00001043 \pm 0.000005618) & (0.002818 \pm 0.0004187) & \mathbf{0} \\ \sim & (0.9961 \pm 0.02997) & (0.05906 \pm 0.002507) \\ \sim & \sim & \mathbf{0} \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} \mathbf{0} & \mathbf{0} & (0.006058 \pm 0.001282)e^{i(-180 \pm 0.04312)^\circ} \\ \sim & (0.9952 \pm 0.04808) & (0.06817 \pm 0.009958)e^{i(36.54 \pm 4.126)^\circ} \\ \sim & \sim & (-0.01568 \pm 0.004455) \end{pmatrix}
\end{aligned} \tag{A22}$$

23.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (0.006004 \pm 0.01056) & (0.09705 \pm 0.05364) & \mathbf{0} \\ \sim & (0.9905 \pm 0.03207) & (0.002462 \pm 0.002079) \\ \sim & \sim & \mathbf{0} \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} (0.033 \pm 0.0138) & (0.0955 \pm 0.0801)e^{i(23.8 \pm 17.2)^\circ} & (0.00552 \pm 0.00151)e^{i(84.1 \pm 42.5)^\circ} \\ \sim & (0.991 \pm 0.0503) & \mathbf{0} \\ \sim & \sim & \mathbf{0} \end{pmatrix}
\end{aligned} \tag{A23}$$

24.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (0.3918 \pm 0.02016) & (0.0002621 \pm 0.00005475) & (-0.486 \pm 0.01299) \\ \sim & \mathbf{0} & \mathbf{0} \\ \sim & \sim & (0.6115 \pm 0.02454) \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} (0.3984 \pm 0.02144) & \mathbf{0} & (-0.5045 \pm 0.02052)e^{i(4.152 \pm 0.3504)^\circ} \\ \sim & \mathbf{0} & (0.007499 \pm 0.000643)e^{i(-95.89 \pm 21.92)^\circ} \\ \sim & \sim & (0.588 \pm 0.03201) \end{pmatrix}
\end{aligned} \tag{A24}$$

25.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (0.01519 \pm 0.009309) & (0.1355 \pm 0.03331) & (0.000218 \pm 0.00002899) \\ \sim & (0.9814 \pm 0.03124) & \mathbf{0} \\ \sim & \sim & \mathbf{0} \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} \mathbf{0} & (0.1574 \pm 0.01544)e^{i(-14 \pm 5.799)^\circ} & (0.006954 \pm 0.001226)e^{i(48.96 \pm 18.69)^\circ} \\ \sim & (0.9714 \pm 0.04805) & \mathbf{0} \\ \sim & \sim & (-0.0005743 \pm 0.0006804) \end{pmatrix}
\end{aligned} \tag{A25}$$

26.

$$\begin{aligned}
M^u &= \chi_u \begin{pmatrix} (0.9797 \pm 0.03049) & (0.1421 \pm 0.02397) & (-0.001423 \pm 0.0003427) \\ \sim & (0.01705 \pm 0.007037) & \mathbf{0} \\ \sim & \sim & \mathbf{0} \end{pmatrix} \\
M^d &= \chi_d \begin{pmatrix} (0.978 \pm 0.0483) & (-0.149 \pm 0.0177)e^{i(-164 \pm 2.03)^\circ} & \mathbf{0} \\ \sim & \mathbf{0} & (0.00669 \pm 0.00149)e^{i(18.6 \pm 14.62)^\circ} \\ \sim & \sim & (-0.000666 \pm 0.00072) \end{pmatrix}
\end{aligned} \tag{A26}$$

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